A tower of right-handed neutrinos and the Swampland

Eduardo Gonzalo Badía

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Based on work with: **G. Obied, M. Montero, C. Vafa and I. Valenzuela.**

Plan for the talk

The leading Swampland tower

Distance/Duality Conjecture. [1]

As $\Lambda_D \to 0$ a tower $m_n^{(D)} \sim |\Lambda_D|^{\alpha_D} M_D^{1-2\alpha_D}$ becomes light, where α_D to be a positive constant.

- Given the smallness of the vacuum energy of our Universe, it is reasonable to expect that a tower either is becoming or became light in the not-so-distant past. [2]
- Today $m_{\text{tow}} = \lambda\, \Lambda^{1/4} \sim 0.01 - 1$ eV.
- Massive KK gravitons induce Yukawa type deviations from Newton's Law which have not been detected and translate into a bound $m_{\text{tow}} \gtrsim 7$ meV (1 single LED is allowed).

[1] D. Lust, E. Palti and C. Vafa '19
[2] M. Montero, C. Vafa and I. Valenzuela '22

The leading Swampland tower

- Why do the scale of neutrino masses and dark energy coincide?
- The SM in a circle would have AdS vacua and be in tension with AdS Swampland conjectures unless there are light fermions.[3]
- These vacua are avoided in the case of Dirac with the lightest lighter than 8 meV.
- Consider a tower of sterile neutrinos with $m_{\text{tow}} \sim 0.01 - 1$ eV.
- Is this even allowed by experiments? What details of the model can soon/easily be tested?
- In this talk: Long Base-Line Neutrino Oscillations.

[3] E.G., L. Ibáñez and I. Valenzuela '21

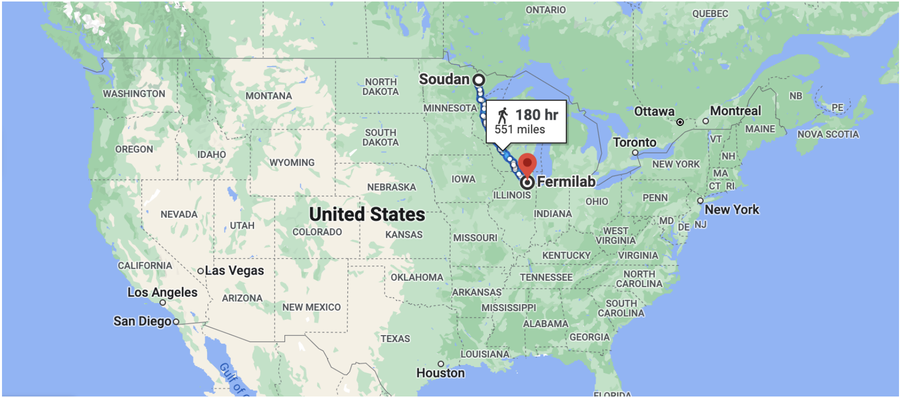

Figure: Example: MINOS experiment

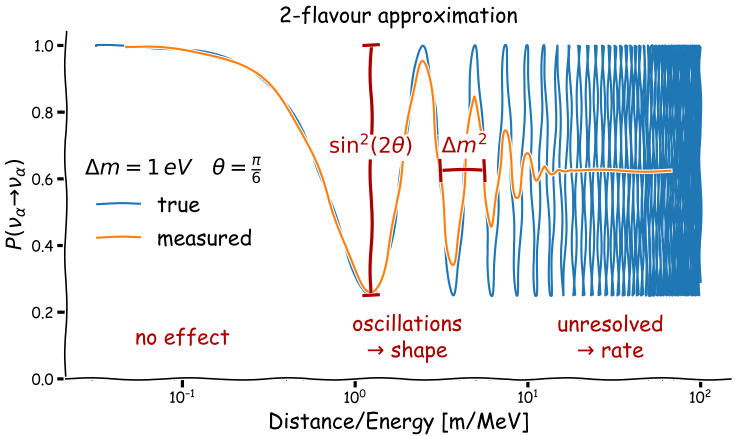

Figure: Taken from 1906.01739. We need sufficient long distance so neutrino oscillations are noticeable but not so much they exceed detector precision.

Description of model

- Start with 3 zero mode active neutrinos and bulk sterile Dirac neutrinos.

$$\mathcal{L}_{4\mathrm{d}} = \overline{\nu}'_\alpha i\partial\!\!\!/ \nu'_\alpha + \sum_{n=1}^{\infty} \{ \overline{N^n_\alpha} (i\partial\!\!\!/ - m_n) N'^n_\alpha \} - \sum_{n=1}^{\infty} \left\{ f^n_{\alpha\beta} \bar{l}_\alpha \frac{\varphi}{v} N'^n_{R,\beta} + \mathrm{h.c.} \right\}$$

- We will not need a detailed description of extra dimensions.
- Neutrino oscillations experiments require at least as starting point a standard 3 flavour oscillation with one mass difference roughly 8.6 meV and another roughly 50 meV (differences are actually squared).
- We should look for cases with 3 mass eigenstates which are mostly active, plus small corrections.

Description of model

$$\mathcal{L}_{4\mathrm{d}} = \overline{\nu}'_\alpha i\partial\!\!\!/ \nu'_\alpha + \sum_{n=1}^{\infty} \left\{ \overline{N^n_\alpha}(i\partial\!\!\!/ - m_n)N'^n_\alpha \right\} - \sum_{n=1}^{\infty} \left\{ f^n_{\alpha\beta}\bar{l}_\alpha \frac{\varphi}{v} N'^n_{R,\beta} + \mathrm{h.c.} \right\}$$

- After symmetry breaking, the mass term is given by

$$\mathcal{L}_{\mathrm{mass}} = nm_0 \left\{ \overline{N'^n_{R,\alpha}} N'^n_{L\alpha} + \mathrm{h.c.} \right\} + f^n_{\alpha\beta}\overline{\nu}'_\alpha N'^n_{R,\beta}.$$

- Notice that the N^0_L decouples.
- First, we diagonalize the flavour index: $\nu'_\alpha = U_{\alpha a}\nu_a$, $N'^n_\alpha = T_{\alpha a}N_a$ where $U^\dagger U = T^\dagger T = \mathbb{I}$, $U^\dagger f^{(n)} T = \hat{f}^{(n)}$.
- U is known as the PMNS matrix.

Description of model

$$\mathcal{L}_{\text{mass}} = \overline{\Psi_{L,a}} \mathcal{M}_a \Psi_{R,a}$$

$$\Psi_{L,a} = (\nu_L, N^1_{La}, N^2_{La}, N^3_{La} \ldots), \qquad \Psi_{R,a} = (N^0_{Ra}, N^1_{Ra}, N^2_{Ra}, N^3_{Ra} \ldots)$$

$$\mathcal{M}_a = \begin{pmatrix} \hat{f}_a^{(0)} & 0 & 0 & 0 & \ldots \\ \hat{f}_a^{(1)} & m_1 & 0 & 0 & \ldots \\ \hat{f}_a^{(2)} & 0 & m_2 & 0 & \ldots \\ \hat{f}_a^{(3)} & 0 & 0 & m_3 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

See [4] for original models.

[4] N. Arkani-Hamed et al '98, K. Dienes at al '99, R. Barbieri et al '00, H. Davoudiasl et al '02, P.A.N. Machado et al '11

Description of model

- Finally, we diagonalize the n index:

$$\Psi^n_{L,a} = (L_a)^n_m \hat{\Psi}^m_{L,a} \qquad \Psi^n_{R,a} = (R_a)^n_m \hat{\Psi}^m_{R,a}$$

- $L^\dagger_a \mathcal{M}_a R = \hat{\mathcal{M}}_a$, where $\hat{\mathcal{M}}_a$ is a diagonal matrix and L_a diagonalizes $\mathcal{M}^\dagger_a \hat{\mathcal{M}}_a$.
- After diagonalization we find:

$$\mathcal{L}_{\text{mass}} = \sum_{n=1}^{\infty} \lambda^n_a m_0 \overline{\hat{\Psi}^n_{L,a}} \hat{\Psi}^n_{R,a}.$$

$$\nu'_\alpha = \sum_{a=1}^{3} U_{\alpha a} \nu_a = \sum_{a=1}^{3} \sum_{n=1}^{\infty} U_{\alpha a} L^{(n)}_{a\,0} \hat{\Psi}^n_{L,a}$$

Description of model

- Start with neutrinos of flavour α and energy E and look for neutrinos of flavour β after they have travelled a distance L, so the quantity we are interested is the probability

$$\mathcal{P}_{\alpha\beta} = \mathcal{P}\left(\nu'_\alpha \to \nu'_\beta\right) = \left|A_{\alpha\beta}\right|^2$$

$$A_{\alpha\beta} = \left\langle \nu'_\beta | \nu'_\alpha(L) \right\rangle = \sum_{a=1}^{3} U_{\alpha a} U^*_{\beta a} \sum_{n=0}^{\infty} \left(L_{a0}^{(n)}\right)^2 e^{i\frac{(\lambda_a^n)^2 m_0^2 L}{2E}}.$$

- We have assumed that neutrinos propagate freely, so we will not deal with i.e. solar neutrinos where matter effects are important.

- We have also assumed that neutrinos are ultra-relativistic.

Description of model

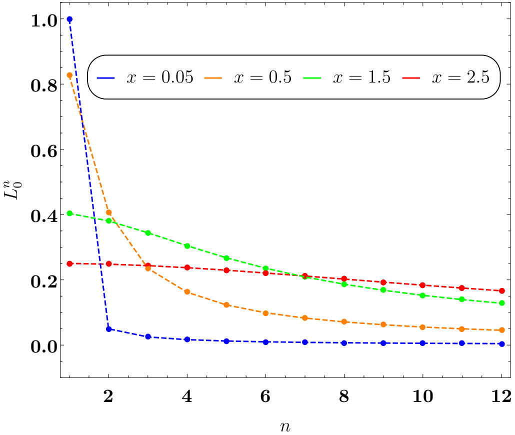

Figure: L_0^n for the $f^n_{\alpha\beta} = f_{\alpha\beta}$ case. We define $x = R^{-1}Y\langle H\rangle$. Only small x is allowed. The most active is always the first excited state with $m_a = f_a - \frac{\pi^2}{12}\frac{f_a^2}{m_0^2}f_a$.

Description of model

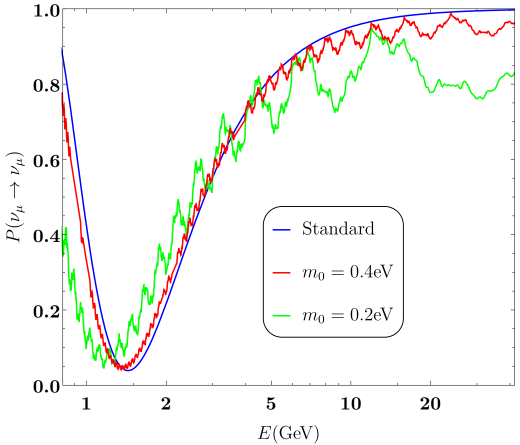

Figure: Example of neutrino oscillation experiment MINOS with 735 km and GeV (muon) neutrinos. First neutrino is massless. More detailed studies (1101.0003) find that $R^{-1} \gtrsim 0.1 - 0.5eV$.

Description of model

- There are other constraints coming from Cosmology ($\sum m_\nu$, N_{eff}) and Astrophysics (missing energy in Supernovae) so it is important to understand possible mechanisms that can suppress the mixing.

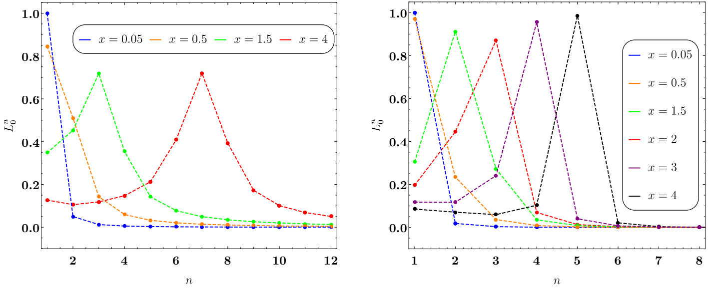

Figure: Left: L_0^n for $f^n_{\alpha\beta} = \frac{1}{n} f_{\alpha\beta}$. Right: L_0^n for $f^n_{\alpha\beta} = e^{-n} f_{\alpha\beta}$. The mostly active state is no longer the first excited state and the mixing is very suppressed.

Description of model

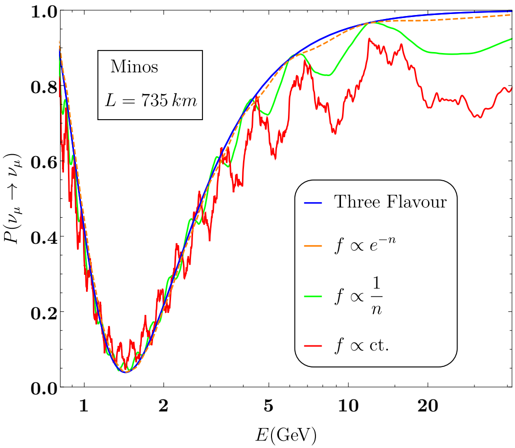

Figure: Neutrino oscillations at MINOS with 735 km and GeV (muon) neutrinos. $m_0 = 0.2$ eV. First neutrino is massless.

What to expect

- Consider dimensional reduction of $U(M)$ SYM theory. Higgs comes from the gauge boson and the neutrinos from the gaugino.

$$S = \int d^{p+1}w \left\{ -\frac{1}{4g^2} \mathrm{Tr}\, F^{MN} F_{MN} + \frac{i}{2g^2} \mathrm{Tr}\, \bar{\lambda} \Gamma^M D_M \lambda \right\}$$

$$A_M = (A_\mu, A_i + \Phi_i) \qquad \Phi_i^{bc} = \sum_n H_{n\,i}^{bc}(x) \times \phi_{n\,i}^{bc}(y)$$

$$\lambda^{ab} = \sum_n \nu_n^{ab}(x) \times \psi_n^{ab}(y) \qquad \lambda^{ca} = \sum_n N_n^{ca}(x) \times \psi_n^{ca}(y)$$

$$Y_{ijk}^{(n,l,m)} = \int d^6y \,\mathrm{Tr} \left(\psi_i^{(n)\dagger} \gamma^\alpha \phi_{j\,\alpha}^{(m)} \psi_k^{(n)} \right)$$

Semi-Realistic Model

- SM particles live on a brane, for example intersecting D6 branes.
- Dark tower lives in a different brane which wraps a large cycle along the extra dimensions.
- We need to go beyond toroidal setup, but still study this case since we can easily compute the Yukawa couplings.
- Consider three stacks of N_α $\alpha = a, b, c$ Dp branes wrapping a T^6 with wrapping numbers m^I_α, $I = 1, 2, 3$.
- Gauge group is broken to $\prod_\alpha U(N_\alpha)$ by flux:

$$F_{z^I \bar{z}^I} = \frac{\pi i}{\mathrm{Im}\tau^I} \begin{pmatrix} \frac{n^I_a}{m^I_a} \mathbb{I}_{N_a \times m_a} & 0 & 0 \\ 0 & \frac{n^I_b}{m^I_b} \mathbb{I}_{N_b \times m_b} & 0 \\ 0 & 0 & \frac{n^I_c}{m^I_c} \mathbb{I}_{N_c \times m_c} \end{pmatrix}$$

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- Gauge group is broken to $\prod_\alpha U(N_\alpha)$ by flux.
- We have a total of $I_{ab} = n_a m_b - m_a n_b$ fermions in the bifundamental $(N_a, \overline{N_b})$.

Semi-Realistic Model

- Torus identifications as $z \sim z + \zeta_\alpha$ where $\zeta_1 = 1$ and $\zeta_2 = \tau$.
- Boundary conditions on the wavefunctions to guarantee that the $U(M)$ vector bundle is well defined:

$$A(z+\zeta_\alpha) = e^{i\omega_a^\alpha U_\alpha} A(z) e^{-i\omega_a^\alpha U_\alpha} + ig^{-1}\left(\partial_\mu e^{i\omega_a^\alpha U_\alpha}\right) e^{-i\omega_a^\alpha U_\alpha}$$

$$\lambda(z+\zeta_\alpha) = e^{i\omega_a^\alpha U_\alpha} \lambda(z) e^{-i\omega_a^\alpha U_\alpha}$$

- Solve Dirac equation and Klein-Gordon equation with fluxes and Wilson lines and compute the integral. See [5-6] for similar computations.

[5] D. Cremades, F. Marchesano and L. Ibáñez, '04
[6] F. Marchesano, P. McGuirk and G. Shiu, '11

Semi-Realistic Model

- We find

$$Y_{(2n)}^{pjk} = \frac{1}{2^n}\frac{\sqrt{(2n)!}}{(n!)}\left(\frac{|I_{ab}|}{|I_{bc}|}\right)^n \times Y_{(0)}^{pjk}$$

$$Y_{(0)}^{pjk} = g\left(\frac{2t_2}{A^2}\right)^{1/2}\left|\frac{I_{ab}I_{ca}}{I_{cb}}\right|^{1/4}\vartheta\begin{bmatrix} -\left(\frac{j}{I_{ca}}+\frac{k}{I_{bc}}\right)/I_{ab} \\ 0\end{bmatrix}(0,|I_{ab}I_{bc}I_{ca}|\,\tau)$$

$$\frac{1}{2^n}\frac{\sqrt{(2n)!}}{(n!)} \sim \frac{1}{2^n}\frac{(4\pi n)^{1/4}}{\sqrt{2\pi n}}\left(\frac{2n}{e}\right)^n\left(\frac{e}{n}\right)^n \sim 1$$

- We find an exponential dependance:

$$Y_{(2n)}^{pjk} \sim \left(\frac{|I_{ab}|}{|I_{bc}|}\right)^n Y_{(0)}^{pjk} = h^{-n}Y_{(0)}^{pjk}$$

where h is larger than 1.

Conclusions

- **Results**:
  1. One can obtain enough suppression if the KK is slightly larger than the $Y\langle H\rangle$, order on 1 eV.
  2. One naturally obtains additional suppression to high KK modes in semi-realistic compactifications where the Yukawa couplings decrease with n.
- **Future Work**:
  1. Can we explain Short Base-Line anomalies (LSND, MiniBooNE, MicroBooNE) with a tower $M_{KK} \sim \mathcal{O}(\text{eV})$ sterile neutrinos?[7]
  2. Cosmological implications: H_0 tension, dark matter [7].

[7] E.G., M. Montero, G. Obied, C. Vafa and I. Valenzuela ’21


THANK YOU
FOR VISITING
THE SWAMP